

### Long Run Economic Growth

*Real GDP per capita* is the key statistic used to track economic growth

$$\text{Real GDP per capita} = \frac{\text{Real GDP}}{\text{Population}}$$

The relationship between the annual growth rate of real GDP per capita and long run changes in real GDP per capita can be summarized using the *rule of 70*, which tells us how long it takes any variable that grows over time to double

$$\text{Number of years for variable to double} = \frac{70}{\text{Growth rate of variable}}$$

**Example 1:** Suppose that real GDP per capita in the US is currently \$2 million, and that this is growing at a rate of 5%. How long will it take for the US to reach real GDP per capita of \$8 million?

The rule of 70 can also be used to compare two countries growing at different rates to one another. In particular, in the fairly common scenario of a smaller economy growing at a fast rate compared to a larger country growing at a slower rate, the rule of 70 can be used to tell us how long it will take for the two economies to even out.

**Example 2:** Suppose that the real GDP per capita in the US is currently \$8 million, and in Ghana it is \$2 million. Furthermore assume that the US economy is growing at 7% per year while Ghana is growing at 14% per year. How long will it take for Ghana's GDP to catch up with the United States?

### Long Run Growth, Productivity, and the Aggregate Production Function

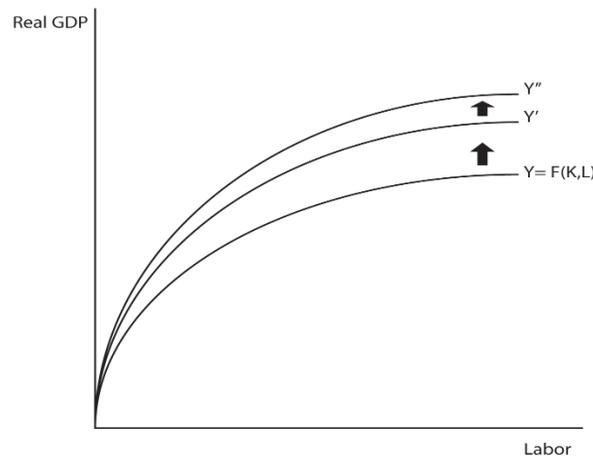
Sustained economic growth only occurs when the amount of output per worker, called, *labor productivity*, increases steadily over time. There are three primary factors which lead to productivity growth over time

- *Increase in physical capital:* *Physical capital* includes goods such as buildings and machines
- *Increase in human capital:* *Human capital* refers to the improvement in labor created by the education and knowledge embodied in the workforce (eg. college degrees)
- *Technological Progress:* this refers to advance in the technical means of the production of goods and services

These ideas are best seen in the *aggregate production function*, which shows how productivity depends on the quantities of physical (human) capital per worker. In drawing the production function, we place *real GDP (Y)* on the *y-axis*, and *physical (human) capital per worker* on the *x-axis*. Physical capital is typically denoted with the variable *K*, while human capital (or labor) is denoted with the variable *L*.

There are several features worth noting on the aggregate production function. First, for any given production function, we assume that the variable that is not featured on the x-axis (in the graph below this is physical capital per worker) is fixed. Assume, for instance, that the function for the lowest line on the graph below is  $Y = F(K, L) = \sqrt{KL}$ . For the first line, we might assume that  $K = 4$ , and thus the equation for the line below is  $Y = \sqrt{4L} = 2\sqrt{L}$ . If  $K$  were to change, say to  $K = 16$ , then the equation would become  $Y = \sqrt{16L} = 4\sqrt{L}$ , which could correspond to  $Y'$  on the graph There are two implications here:

- If the variable on the x-axis changes, this is a *movement along* the production function
- If the variable which isn't on the x-axis changes, it is a *shift* of the production function



Second, notice the shape of the production function. Typically, production functions are *concave*, meaning that their slopes decrease as you increase the variable on the x-axis. In economics, we say this is due to the *law of diminishing returns*. This law states that when you increase one input (labor/human capital in the graph above), while holding the other input fixed (human capital in this graph above), those increases yield smaller gains in terms of real GDP. The intuition here can be illustrated with an example. Suppose you have a pizza shop with 5 ovens. Initially you only have one person working in the shop. Initially, as you hire more people and/or increase their knowledge of how to produce pizzas, they can experience large gains in productivity. However, as you hire more people, if you do not build more ovens (physical capital), there are “too many cooks in the kitchen,” and they begin to get in each other’s way, limiting the benefit to hiring more people.

Finally, the production function can be used to illustrate labor/capital productivity

$$\text{Labor (or Capital) Productivity} = \frac{Y}{L \text{ (or } K)}$$

A couple of things to note here. First, it is possible to directly calculate productivity using the formula above, if you are given the production function, and the amount of labor and capital being used.

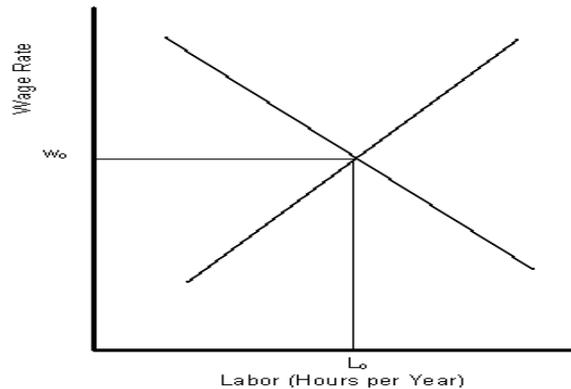
Secondly, you can illustrate how productivity changes graphically by drawing a line from the origin through the current point of production on the production function. The steeper the line, the higher productivity is.

**Example 3:** Illustrate graphically how a change in labor affects the productivity of labor and capital

### The Labor Market

The labor market functions like a most markets from Econ 101. However, there are a couple of key differences. First, on x-axis we put the quantity of labor, often denoted with the variable  $L$ , instead of  $Q$ . On the y-axis we have the *wage rate*, denoted with  $W$ , instead of  $P$ . Furthermore, while we have a downward sloping demand curve/upward sloping supply curve as usual, note that the demand in this market is coming from *firms looking to hire labor*, while supply is provided by *individuals looking for employment*. As a result, unemployment can be modeled as any situation where the *quantity demanded*

is greater than the quantity supplied, which is the case then the wage rate is above the equilibrium wage rate.



Tutorial: Solving problems using the labor market and the aggregate production function

Typically, problems asking you to use the labor market and the aggregate production function give you a labor supply/demand function, a production function, and an initial level of capital. Here is how to solve a standard problem given this information

*Step 1: Determine the equilibrium quantity of labor*

As in most supply and demand situations, you can determine the equilibrium quantity of labor by setting the labor supply and demand equations equal to each other

*Step 2: Plug the equilibrium level of labor, along with the given amount of capital, into the production function*

*Step 3 (if applicable): Use the determined level of production along with the equilibrium level of labor (or capital) to determine labor (or capital) productivity.*

There are some notable modifications to these problems to note. One such modification involves how a shift in labor demand (or supply) will affect equilibrium labor, production, and productivity. If you are given the actual functions, you simply need to make the appropriate shift (as discussed in Handout 1), repeat the steps above, and make any necessary comparisons (if you are asked to do this graphically, and without functions, a useful tip would be to carefully mark all of your graphs, and then remember to use the “straight line trick” outlined earlier. Finally, remember that the minimum wage in this market acts as a *price floor*, meaning that there is an excess supply of labor, which we refer to as *unemployment*. In this case, you need to use the quantity of labor demanded in your calculations.

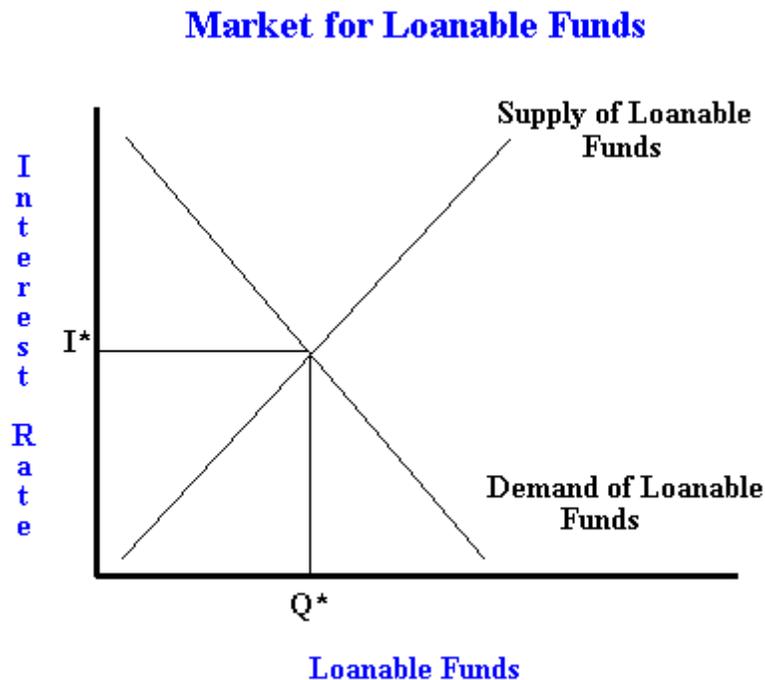
**Example 4:** Using the following information, please calculate the equilibrium level of labor, the level of production, and labor/capital productivity

$$\begin{aligned}
 F(K, L) &= \sqrt{KL} \\
 K &= 4 \\
 W &= 72 - 2L^D \\
 W &= 8 + 2L^S
 \end{aligned}$$

### Savings-Investment Spending Identity

- Recall that  $Y = C + I + G + NX$
- Now suppose that we are in a closed economy (i.e.  $NX=0$ )
- Thus, we have  $Y = C + I + G$  (1)
- We can also think of Savings as  $S = Y - C - G$  (2)
- If we substitute our definition of  $Y$  from (1) into (2), we are thus left with the following:
 
$$S = Y - C - G = I + (C + G) - (C + G) = I$$
- Further, we can split savings into two components:
  - $S_{\text{Private}} = Y - C - (T - TR)$
  - $S_{\text{Public}} = T - G - TR$
  - $S_{\text{National}} = S_{\text{Private}} + S_{\text{Public}}$

### Market for Loanable Funds



- Suppliers in this market are **households saving money for future consumption**
- Demand comes from **firms looking to make investments**
- The “price” in this market is the **real interest rate**
  - The “law of demand” and “law of supply” apply here: as the real interest rate rises firms are less likely to borrow money and households are more likely to save money
- Shifts in Demand include: Perceived changes in business opportunities and changes in government spending
- Shifts in Supply include: Changes in private behavior and in net capital flows

### **Net Capital Inflows and Savings in an Open Economy**

In an open economy, savings do not need to be spent on investment projects within the same country in which the savings are generated. As a result, a country can now receive an *inflow* of funds—foreign dollars which finance projects in that country. Countries can also generate *outflows* of funds—domestic savings that finance investment spending in a different country. The *net capital inflow* into a country describes the net effect of the international inflows and outflows for a given country

$$\text{Net Capital Inflows} = NCI = IM - X$$

Integrating this formula with our earlier savings identity, we get

$$I = GDP - C - G + (IM - X) = S_{National} + NCI$$

### **Crowding Out:**

The *crowding out effect* states that an increase in the Government deficit will reduce private investment and consumption. An increase in the government deficit in a closed economy leads to an increased demand for loanable funds. This increases the equilibrium interest rate, leading to decreased investment by private businesses

### **Practice Problems**

Use the following information to answer the next THREE questions.

Suppose an economy's aggregate production function is given by the equation:

$$Y = 10\sqrt{KL}$$

where Y is real GDP, K is units of capital and L is units of labor. Furthermore suppose you know that the labor market in this economy can be described by the following demand and supply curves where W is the wage rate:

Demand for Labor:  $L = 200 - 10W$

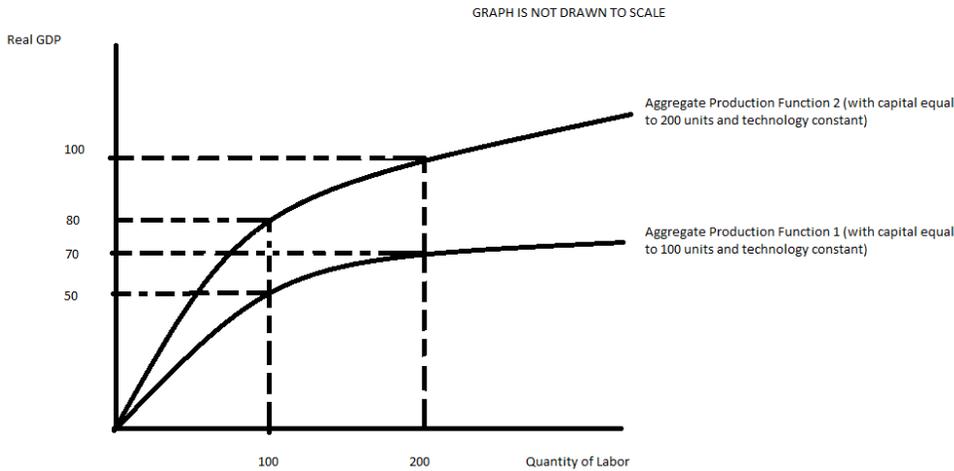
Supply of Labor:  $L = 10W$

Assume capital is initially equal to 400 units in this economy.

- Given the above information, full employment GDP for this economy is equal to
  - 2000
  - 200
  - 1000
  - 10,000
- Suppose the amount of capital increases to 625 units. Holding everything else constant, labor productivity will increase to
  - 0.25 units of output per unit of labor
  - 0.3 units of output per unit of labor
  - 25 units of output per unit of labor
  - 30 units of output per unit of labor

3. Suppose capital is at its initial level. Now suppose the government imposes a minimum wage in this labor market equal to \$17.50 per unit of labor. Given this information and holding everything else constant
- Real GDP = 100 units of output
  - Real GDP = 1000 units of output
  - Real GDP = 500 units of output
  - Real GDP = 5000 units of output
4. Suppose technology improves in an economy. Holding everything else constant
- Real GDP will increase in this economy.
  - Labor productivity will increase in this economy.
  - Capital productivity will increase in this economy.
  - Answers (a), (b), and (c) are all true statements.
  - Answers (a), (b) and (c) are all false statements.
5. Suppose the labor market is initially in equilibrium. If the demand for labor increases at every wage rate then holding everything else constant
- Real GDP will increase, the equilibrium wage rate will increase, and labor productivity will fall.
  - Real GDP will increase, the equilibrium wage rate will increase, and labor productivity will increase.
  - Real GDP will decrease, the equilibrium wage rate will increase, and labor productivity will fall.
  - Real GDP will increase, the equilibrium wage rate will decrease, and labor productivity will fall.
6. Suppose real GDP in 2010 is \$5 billion and in 2011 real GDP increases by 2%. Suppose population in 2010 is 1 million people and in 2011 population increases by 10,000 people. From this information you can conclude
- Real GDP per capita was constant between 2010 and 2011.
  - Real GDP per capita increased between 2010 and 2011.
  - Real GDP per capita decreased between 2010 and 2011.
  - This economy is not growing fast enough to absorb the increase in population.
7. Real GDP per capita in Eurolandia in 2013 is \$20,000 while in Utopia real GDP per capita in 2013 is \$15,000. Both countries are projected to have steady growth rates of real GDP per capita over the next 200 years: the growth rate in Eurolandia is projected to be 5% while the growth rate in Utopia is projected to be 10%. Given this information and the Rule of 70
- By 2041 real GDP per capita in Utopia will be approximately three times greater than real GDP per capita in Eurolandia.
  - By 2027 real GDP per capita in Utopia will exceed real GDP per capita in Eurolandia.
  - In 2020 real GDP per capita in Utopia will be approximately equal to \$30,000.
  - Given the above information, answers (a), (b) and (c) are true statements.
  - Given the above information, answers (a) and (c) are true statements.

8. Use the following graph of an economy’s aggregate production function to answer this set of questions.



- a. Suppose this economy has 100 units of capital and 100 units of labor. Holding everything else constant, the value of labor productivity is \_\_\_\_\_.
  - b. Suppose this economy increases its level of capital to 200 units while maintaining labor usage at 100 units. Given this information and the above graph verbally explain what will happen to labor productivity.
  - c. Given the information in (b) calculate the new value of labor productivity.
  - d. Given the information in (b) provide a graph in the space below illustrating the relationship between the level of capital (measure this on the x-axis) and the level of real GDP (measure this on the y-axis). Provide the coordinates for at least two points in this graph. (Hint: Plot the curve of production function when the level of labor is 100)
  - e. Given the information in (b), verbally describe what happens to capital productivity.
9. There is a government budget deficit if
- A)  $T - \text{Transfers} > G$ .
  - B)  $G < T$ .
  - C)  $G < TR$ .
  - D)  $\text{Transfers} + G > T$
10. If technological change increases the profitability of new investment for firms, then the \_\_\_\_\_ curve for loanable funds will shift to the \_\_\_\_\_ and the equilibrium real interest rate will \_\_\_\_\_.
- A) demand; right; rise
  - B) supply; left; rise

- C) supply; right; fall  
D) demand; left; fall

11. In a closed economy,

- a)  $Y = C + G + NX$   
b)  $Y = C + I + S$   
c)  $Y = C + I + T$   
d)  $Y = C + I + G$

12. A fall in the market interest rate makes any investment project:

- a) More profitable whether the funds were borrowed or came from retained earnings  
b) More profitable only if the funds were borrowed  
c) Less profitable whether the funds were borrowed or came from retained earnings  
d) Less profitable of the funds were borrowed and more profitable if it came from retained earnings

13. Interest rates and planned investment spending:

- a) Exhibit a negative relationship  
b) Have no relationship, since planned investment is fixed  
c) Have no relationship if the firm has retained earnings  
d) Have a positive relationship

14. An increase in the level of business opportunity will general:

- a) Shift the loanable funds demand curve to the right  
b) Cause a movement either up or down the loanable funds demand curve  
c) Not change the loanable funds demand curve  
d) Shift the loanable funds demand curve to the left

15. Suppose that the market for private loans is given by the following equations:

$$\text{Demand: } i = 20 - \frac{1}{5}Q$$

$$\text{Supply: } i = Q - 10$$

where  $Q$  is the quantity of loans and  $i$  is the interest rate (the price of loans). Initially, only private individuals demand loans. Then, the government decides to run a budget deficit such that the government's demand for loans is given by the equation:

$$\text{Demand: } Q = 12$$

Initially, the equilibrium quantity of private loans was \_\_\_\_\_ and the interest rate was \_\_\_\_\_. After the government begins to run it's deficit, the new quantity of private loans is \_\_\_\_\_ and the new interest rate is \_\_\_\_\_.

- a) 25; 15; 37; 27  
b) 15; 17; 25; 15  
c) 25; 15; 15; 17  
d) 15; 17; 23; 13