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Economics 712
Macroeconomic Theory
Spring 2016

Final Examination

Instructions: This is a 75 minute exam with worth a total of 100 points. Point values on each part are marked. **Allocate your time wisely.** In order to get full credit, you must give a clear, concise, and correct answer, including all necessary explanations and calculations. Notes, books, and calculators are not permitted.

1. **[50 points]** Consider a representative agent exchange economy with money, where the aggregate endowment Y_t is governed by an exogenous process:

$$\log \frac{Y_t}{Y_{t-1}} = \mu + \sigma W_t \tag{1}$$

where $\mu \geq 0$ is the mean growth rate, and W_t is an i.i.d. standard normal endowment shock. Preferences over consumption c_t and real money balances $m_t = M_t/P_t$ are:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} + v(m_t) \right],$$

where v is strictly increasing, strictly concave, and differentiable. The agent can trade in a stock (claim to the endowment stream) with price S_t, a risk-free real bond (paying one unit of real goods) with price $1/R_t$, and a risk-free nominal bond (paying one unit of nominal goods with real value P_t/P_{t+1}) with price $1/I_t$.

Denote household wealth x_t and suppose the agent is endowed with the stock and the initial money: $x_0 = S_0 + M_0/P_0$. The agent then chooses his consumption c_t, real money holdings m_t, holdings of the real bond α_{bt}, the nominal bond α_{B_t} and the stock α_{St}. The agent's wealth is then:

$$x_t = \alpha_{bt} + \alpha_{Bt} + \alpha_{St} + m_t$$

which satisfies the budget constraint:

$$x_{t+1} \quad = \quad x_t - c_t + \alpha_{b_t}(R_t - 1) + \alpha_{Bt}(I_t - 1) + \alpha_{St} r_t^s + \frac{M_t^s}{P_t} - m_t$$

where $r_t^s = (Y_t + S_t)/S_{t-1} - 1$ is the return on the stock.

Postmortem: This budget constraint appeared on the exam, but it is incorrect. See solution for details.

(a) **[20 points]** Find the agent's optimality conditions, then impose the equilibrium conditions (with nominal and real bonds in zero net supply) to characterize equilibrium prices and interest rates.

(b) **[10 points]** Given the specification for the endowment process, solve explicitly for the net real interest rate $r_t = \log(R_t)$ and describe how it depends on the growth and volatility of output and the agent's preferences.

(c) **[5 points]** We will solve for equilibria of the form $P_t = Y_t^a$ for some a. Define $\pi_t = \log E_t(P_{t+1}/P_t)$ as the net expected inflation rate. Show that a given π_t is (typically) consistent with two values of a.

(d) **[5 points]** Solve for equilibrium nominal interest rate $i_t = \log(I_t)$ in this class of equilibria.

(e) **[10 points]** Suppose that monetary policy pegs a constant interest rate $i_t = \bar{i}$. Show that if $\sigma = 0$ there is a unique equilibrium, but if $\sigma > 0$ there are two equilibria. Interpret your answer in terms of the Fisher equation and inflation risk.

2. **[30 points]** Consider a continuous time search model with variable intensity. That is unemployed workers earn benefits z and choose an intensity level s which has (monetary) utility costs $c(s)$ which are increasing and convex, but increases the likelihood of finding a job $q(s)$ where q is increasing and concave. When employed, a worker earns a constant wage w and the job is subject to destruction at rate λ.

   (a) **[20 points]** Write down the Hamilton-Jacobi-Bellman equations determining the value $U(s)$ of an unemployed worker who searches with intensity s and W of an employed worker. Find the steady state values of $U(s)$ and W.

   (b) **[5 points]** Characterize the optimal choice of s for a currently unemployed worker (taking as given his search intensities in future unemployment spells), assuming $q'(s) = q(s)/s$.

   (c) **[5 points]** How does the optimal search intensity in a steady state respond to an increase in the wage w?

3. **[20 points]** Answer the following:

   (a) **[7 points]** Why is inflation costly in the New Keynesian model?

   (b) **[7 points]** What is the equity premium puzzle and why can't it be resolved by higher γ with preferences $u(c) = c^{1-\gamma}/(1-\gamma)$?

   (c) **[6 points]** What is a time consistency problem and how does it arise in Ramsey optimal taxation?