Econ 714: Handout 1 - Solution [1]

1 Mortensen-Pissarides model

Compared to Pissarides, job destruction rate is endogenous. Each job has productivity px, where x is idiosyncratic. New x arrives at Poisson rate λ, drawn from distribution G on $[0,1]$. Initial draw is $x = 1$.

Value of a job is now $J(x)$. If $J(x) \geq 0)$ job kept, if $J(x) < 0$ destroyed. Reservation productivity R such that $J(R) = 0$.

Job destruction rate: $\lambda G(R)(1-u)$. Job creation: $m(u,v) = \theta q(\theta)u$, where $\theta = v/u$ is market tightness. Unemployment flow: $\dot{u} = \lambda G(R)(1-u) - \theta q(\theta)u$

Steady state (Beveridge curve):

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)} \tag{BC}$$

Value functions for the firm:

$$rV = -pc + q(\theta)(J(1) - V) \tag{FV}$$

$$rJ(x) = px - w(x) + \lambda \left[\int_R^1 J(s)dG(s) - J(x) \right] \tag{FJ}$$

Value functions for the worker:

$$rU = z + \theta q(\theta)(W(1) - U) \tag{WU}$$

$$rW(x) = w(x) + \lambda \left[\int_R^1 W(s)dG(s) + G(R)U - W(x) \right] \tag{WW}$$

Worker's share of surplus (Nash bargaining):

$$W(x) - U = \beta[W(x) - U + J(x) - V] \tag{NB}$$

Zero profit: $V = 0$.
Exogenous variables: $\lambda, G, m, p, c, z, r, \beta$.
Endogenous variables: $R, \theta, u, v, w, V, J, U, W$.

1.1 Solving the model

1. Wage equation:

$$w(x) = z(1-\beta) + \beta p(x + c\theta) \tag{w}$$

From (FV) and $V = 0$:

$$J(1) = \frac{pc}{q(\theta)}$$

[1] By Anton Babkin. This version: January 31, 2016.

Substitute into (NB) with $x = 1$:

$$W(1) - U = \frac{\beta}{1-\beta}\frac{pc}{q(\theta)}$$

Plug into (WU):

$$rU = z + \theta\frac{\beta}{1-\beta}pc$$

Multiply (WW) by $1 - \beta$ and subtract (FJ) multiplied by β. Substitute out $W(x)$ and $J(x)$ using (NB) and get:

$$w(x) = \beta px + r(1-\beta)U$$

Use previously found expression for rU to derive (w).

2. Job creation:

$$(1-\beta)\frac{1-R}{r+\lambda} = \frac{c}{q(\theta)} \tag{JC}$$

Plug (w) into (FJ):

$$(r+\lambda)J(x) = (1-\beta)(px - z) - \beta pc\theta + \lambda\int_R^1 J(s)dG(s) \tag{1}$$

Evaluate (1) at $x = R$ and subtract resulting equation from (1), using $J(R) = 0$:

$$(r+\lambda)J(x) = p(1-\beta)(x - R) \tag{2}$$

Evaluate at $x = 1$ using $J(1) = \frac{pc}{q(\theta)}$ and rearrange to get (JC).

3. Job destruction:

$$\frac{\beta}{1-\beta}c\theta = R - z/p + \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s) \tag{JD}$$

Use (2) to substitute $J(s)$ under integral in (1):

$$(r+\lambda)J(x) = (1-\beta)(px-z) - \beta pc\theta + \frac{\lambda}{r+\lambda}p(1-\beta)\int_R^1 (s-R)dG(s)$$

Evaluate at $x = R$ and divide by $p(1-\beta)$ to get (JD).

4. Solve (JC) and (JD) for R and θ, then use (BC) to solve for u and v.

We can't derive closed form solutions, but can argue that solution is unique since (JC) is decreasing and (JD) is increasing in (θ, R) space.

Graphs can be used to do comparative statics. For example, if p increases, (JD) curve shifts to the right, so R decreases, θ increases. From (BC), u is decreasing and from definition of θ, v must increase.

2 Problem - McCall model[2]

Consider a variation on the basic sequential search model in which there is wage growth. Agents are risk neutral and seek to maximize:

$$E\sum_{t=0}^{\infty}\beta^t y_t$$

where y_t is income in period t, which comes either from work or unemployment benefis, and $0 < \beta < 1$. Suppose that there are no separations and each unemployed worker is sure to receive an offer upon searching. If the wage offer is w in the first period, then the wage is $w_t = \phi^t w$ after t periods on the job, where $\phi > 1$ and $\phi\beta < 1$. The initial wage offer is drawn from a constant distribution $F(w)$. Unemployed workers earn a constant benefit of z.

1. Write down an unemployed worker's Bellman equation and characterize his optimal decision strategy.

   Start with value of an employed worker with wage w:

   $$W(w) = w + \beta\phi w + \beta^2\phi^2 w + ... = \frac{w}{1-\beta\phi}$$

   Value of an unemployed:

   $$U = z + \beta\int_0^{\infty}\max\{U, \frac{w}{1-\beta\phi}\}dF(w)$$

   Optimal decision is to accept if $w > w_R$ and reject if $w < w_R$, where at w_R worker is indifferent: $U = W(w_R) = \frac{w_R}{1-\beta\phi}$. Split integral in two parts:

   $$\frac{w_R}{1-\beta\phi} = z + \beta\int_0^{w_R}\frac{w_R}{1-\beta\phi}dF(w) + \beta\int_{w_R}^{\infty}\frac{w}{1-\beta\phi}dF(w)$$

   Add and subtract $\beta\int_{w_R}^{\infty}\frac{w_R}{1-\beta\phi}dF(w)$ to the RHS:

   $$\frac{w_R}{1-\beta\phi} = z + \beta\frac{w_R}{1-\beta\phi} + \frac{\beta}{1-\beta\phi}\int_{w_R}^{\infty}(w-w_R)dF(w)$$

   Rearrange and multiply by $(1-\beta\phi)$:

   $$(1-\beta)w_R - z(1-\beta\phi) = \int_{w_R}^{\infty}(w-w_R)dF(w) \quad (3)$$

   We can't solve it explicitly, but can characterise solution by plotting LHS and RHS as functions of w_R. LHS is clearly increasing. To show that RHS is decreasing in w_R, we need a negative derivative.

[2] August 2012 macro prelim

We will use the Leibniz's rule:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{a(t)}^{b(t)} f(x,t)\,\mathrm{d}x\right) = \int_{a(t)}^{b(t)} \frac{\partial f}{\partial t}\,\mathrm{d}x \,+\, f\big(b(t),t\big)\cdot b'(t) \,-\, f\big(a(t),t\big)\cdot a'(t)$$

Applying to the RHS yields (w_R plays the role of t, $dF(w) \equiv f(w)dw$):

$$-\int_{w_R}^{\infty} dF(w) + \lim_{b\to\infty}[(b - w_R)f(b)\cdot 0] - (w_R - w_R)\cdot 1 = -(1 - F(w_R)) < 0$$

2. Suppose that there are two economies $i = 1, 2$ that differ in their wage growth rates, with $\phi_1 > \phi_2$ (both ϕ_i still satisfy $1 < \phi_i < 1/\beta$). How do the decision strategies differ across economies?

   From (3) it is clear that increase in ϕ shifts the upward sloping LHS curve up, so solution w_R must be lower, i.e. $w_{R1} < w_{R2}$.

   Intuitively, if wage grows faster once employed, it is better to start working earlier, so reservation wage is lower.