

## Econ 714: Handout 2 <sup>1</sup>

### 1 McCall with savings<sup>2</sup>

Recall the McCall search model: when a worker is unemployed, he receives a constant benefit  $z$ , searches for a job, and receives an offer  $w$  drawn from a distribution  $F(w)$ . Wage offers are i.i.d. over time, and if a worker accepts he works forever at that wage  $w$ . Now suppose that workers (both employed and unemployed) are able to borrow or save in an asset  $a$  yielding fixed gross return  $R$ . The flow budget constraint is thus:

$$c_t + a_{t+1} = Ra_t + y_t$$

where here  $y_t$  is the workers income, which is either  $z$  if he is unemployed or  $w$  if he is employed. Assume there is a borrowing constraint  $a_t \geq \underline{a}$  which never binds. Workers seek to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where  $0 < \beta < 1$  and  $u$  is bounded and satisfies  $u' > 0$ ,  $u'' < 0$ .

1. Write down the Bellman equations characterizing the values of employed and unemployed workers.
2. Characterize an unemployed workers decisions for responding to job offers, consumption, and savings as sharply as you can.

### 2 Lucas tree<sup>3</sup>

Suppose that a representative agent has preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

over the single non-storable consumption good (fruit), where  $\gamma > 0$ . Her endowment of the good is governed by a Markov process with transition function  $F(x, x')$ .

1. Define a recursive competitive equilibrium with a market in claims to the endowment process (trees).
2. When the consumer has logarithmic utility,  $u(c) = \log(c)$ , what is the equilibrium price/dividend ratio of a claim to the entire consumption stream? How does it depend on the distribution of consumption growth?
3. Suppose there is news at time  $t$  that future consumption will be higher. How will prices respond to this news? How does this depend on the consumers preferences (which could be CRRA, not necessarily log)? Interpret your results.
4. Suppose that the endowment process is characterized by lognormal growth. That is,  $x_{t+l} = x_t \exp(\xi_{t+1})$ , where  $\xi_t \sim N(\mu, \sigma^2)$  i.i.d. What is one-period risk free interest rate? How does it depend on the preference parameter  $\gamma$ ? Interpret your results.
5. Consider an option which is bought or sold in period  $t$  and entitles the current owner to exercise the right to buy one tree in period  $t+1$  at the fixed price  $\bar{p}$  specified at date  $t$ . (The buyer may choose not to exercise this option.) Find a formula for the price of this option in terms of the parameters describing preferences and endowments.

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<sup>1</sup>By Anton Babkin. This version: February 5, 2016.

<sup>2</sup>Spring 2014 midterm

<sup>3</sup>August 2012 macro prelim