Econ 714: Handout 3 [1]

1 Term structure of interest rates

Consider the following environment. The economy has a Lucas tree that issues an uncertain dividend s_t each period, which follows the Markov transition function $Q(s, s')$. Define the risk-free (gross) interest rate between periods t and $t + j$ as R_{jt} (corresponding price $q_{jt} = 1/R_{jt}$), measured in units of the $t + j$ consumption good per the time t consumption good. Assume that all bonds are zero-coupon. Let b_{jt} denote the holdings of the bond with maturity j at time t, and let a_t denote holdings of the tree. Suppose the representative agent has time-additive utility such that $u' > 0, u'' < 0$, and $\lim_{c \to 0} u'(c) = \infty$.

1. Assume that the only bonds in the market are of 1- and 2-period maturity, and each is in zero net supply. Write down the agent's Bellman equation and the relevant budget constraint and market clearing conditions (i.e. define an equilibrium).

2. Solve for the prices of all three assets. Does the relationship

$$q_{2t} = q_{1t}\mathbb{E}_t(q_{1t+1})$$

   hold? Why or why not? If not, what changes can we make to the model so that it does?

3. The **term structure** is the collection of yields to maturity across dates of maturity for all bonds within a certain class. The **yield to maturity** of a bond is the return earned by an investor who purchases the bond today and holds it to maturity, collecting all coupon payments along the way and the return of principal at the end. Assume that the dividend process is i.i.d. to ease the computation. Compute the yields to maturity and therefore term structure for the risk-free assets in this economy. Under what conditions is the yield curve increasing?

4. BONUS (an easy one): Generalize your answer to part 3 to the case when the economy has bonds with maturities 1,2,...j.

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2 Burrowed are?[2]

An economy consists of two types of consumenrs indexed by $i = 1, 2$. There is one nonstorable consumption good. Let (e_t^i, c_t^i) be the endowment, consumption pair for consumer i in period t. Both consumers have preferences ordered by

$$U^i = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \log(c_t^i)$$

The endowment streams of the two consumers are governed by two independent 2-state Markov chains $s_t \in \{0, 1\}$ with transition matrix P_s and $a_t \in \{0, 1\}$ with transition matrix P_a, where $P_{s,12} = Pr\{s_{t+1} = 1 | s_t = 0\}$ and so on. Suppose that each chain is stationary and that the initial states (s_0, a_0) are drawn from the stationary distribution, so that $Pr\{s_0 = 0\} = \overline{\pi}_s$ and $Pr\{a_0 = 0\} = \overline{\pi}_a$. Consumer 1 has endowment $e_t^1 = a_t + s_t$, while consumer 2 has endowment $e_t^2 = a_t + 1 - s_t$.

(a) Write out explicitly the preferences U^i of a consumer in terms of the history of the states and their associated probabilities.

(b) Define a competitive equilibrium for this economy.

(c) Characterize the competitive equilibrium for this economy, calculating the prices of all Arrow-Debreu securities. How does the allocation vary across the (s_t, a_t) states? Across time? Across consumers?

(d) Now consider an economy with sequential trading in Arrow securities, one-period ahread claims to contingent consumption. How many Arrow securities are there? Compute their prices in the special case $\beta = 0.95$, $P_{s,11} = 0.9$, $P_{s,22} = 0.8$, $P_{a,11} = 0.8$, $P_{a,22} = 0.7$.

(e) Using these same parameters, in each state what is the price of a one-period ahead riskless claim to one unit of consumption?

[2]Spring 2016 problem set