Econ 714: Handout 5 [1]

1 Optimal taxation with private information[2]

The model economy has two periods and a unit measure of agents. Each angent is endowed with y_1 units of the single consumption good in period 1. Consumption can be stored from one period to the next.

In period 2, agents can exert effort to generate consumption. Measure p_H of the agents are highly skilled, and one unit of their effort generates θ_H units of consumption. Measure p_L are low-skilled. For them, one unit of effort generates θ_L units of consumption, where $\theta_L < \theta_H$. $p_H + p_L = 1$.

Agents' utility function is

$$U(c_1, c_2, l_2) = u(c_1) + \mathbb{E}[u(c_2) - v(l_2)]$$

where c_t is consumption in period t and l_t is effort in period t. $u' > 0, u'' < 0, v' > 0, v'' > 0$.

Timing is as follows. In period 1 agents choose consumption c_1 and savings S. In the beginning of period 2 idiosyncratic productivity θ is realized, with $Pr(\theta = \theta_H) = p_H$ and $Pr(\theta = \theta_L) = p_L$. Then the agent chooses c_2 and l_2. Productivity and effort are private invormation, but savings and output $y = \theta l$ are publicly observed.

1. Characterize first-best allocation. Explain why it is not attainable under private information.

2. Formulate social planner's problem with private information. Characterize socially optimal allocation: derive the Euler equation and show that there is a wedge between intertemporal MRS and MRT.

3. Formulate agent's problem in a decentralized environment, where government imposes a tax $\tau(S, y_2)$ in period 2. Assume that $p_L = p_H = 1/2$. Show that socially optimal marginal tax on savings $\tau_S(S, y_2)$ depends on y_2 and $\tau_S(S, y_2^H) < \tau_S(S, y_2^L)$.

[1] By Anton Babkin. This version: February 29, 2016.

[2] Adapted from Kocherlakota (2004) Wedges and Taxes, AER Papers and Proceedings.

2 Cash-in-advance[3]

Consider a cash-in-advance model in which there are two types of goods: c_1 requires money M_t to purchase, while c_2 can be purchased on credit. The two goods are technologically equivalent, as the endowment e_t can be converted one-for-one into either of them, so $e_t = c_{1t} + c_{2t}$. Suppose that e_t follows a Markov process with transition density $Q(e'|e)$. A representative agent in this economy thus solves:

$$\max_{\{c_{1t}, c_{2t}, M_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t})$$

subject to the budget constraint:

$$P_t c_{1t} + P_t c_{2t} = P_t e_t + M_t - M_{t+1},$$

and the cash-in-advance constraint:

$$P_t c_{1t} \leq M_t.$$

1. Write down the Bellman equation for the representative household and find the optimality conditions.

2. Consider a steady state equilibrium in which the endowment is constant $e_t = e$, the money supply grows at a constant rate: $M_{t+1} = \mu M_t$, and real balances M_t/P_t are constant. What is the minimal level of μ that will support a steady state monetary equilibrium? Is such equilibrium Pareto efficient?

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[3]Spring 2013 problem set.