Econ 714: Handout 10 [1]

1 Investment with adjustment costs and taxation[2]

Firm owns productive capital K_t that generates output $F(K_t)$ $(F_K > 0, F_{KK} \leq 0)$ and evolves according to $K_{t+1} = (1-\delta)K_t + I_t$. Output can be transformed into investment goods I_t one for one, but investment entails convex adjustment costs of $\Psi(I_t, K_t)$: $\Psi_I > 0, \Psi_{II} > 0, \Psi_K < 0, \Psi_{KK} > 0$ and $\Psi_I(\delta K, K) = 0$, i.e. marginal adjustment cost is zero when investment just replaces depreciating capital. One commonly used functional form is $\Psi(I,K) = \frac{psi_0}{2K}(I-\delta K)^2$.

Corporate profits are subject to taxation characterized by the following rules:

- Operating profit is taxed at rate τ.
- Depreciation allowance. Capital expenditures can be deducted from taxable profit at depreciation schedule D_s, where $s = 0, 1, 2, ...$ is the number of periods since the capital was installed. Assume that D_s follows a simple linear rule: every period a constant fraction δ of the *initial* value of capital can be deducted, i.e. for tax purposes capital fully depreciates after $1/\delta$ periods.
- Investment tax credit: A fraction κ of capital expenditures can be subtracted from the tax bill immediately.
- Assume that the above rules symmetrically apply if before-tax profit is negative, in which case firm gets a refund.

Firm starts with initial level of capital K_0 and is choosing optimal investment policy to maximize present value of after-tax profits, discounted at interest rate r, $V(K_0)$.

1. Formulate firm's decision problem. Pay attention to all the tax rules.
2. Denote the shadow value of capital by q_t. Write down the Lagrangian and characterize firm's optimal investment policy.
3. Assume that firm starts at the steady state. Use phase diagram to describe firm behavior after an unanticipated policy change that allows to depreciate capital for tax purposes at a faster rate $\hat{\delta} > \delta$ (depreciation rule is still linear, and physical depreciation is not affected).
4. (Hayashi theorem). Show that if $F(K)$ and $\Psi(I,K)$ are linearly homogenuous, then Tobin's marginal q and average $Q \equiv V/K$ are related as $q = Q + \hat{A}$, where $\hat{A}$ is a constant.
5. Describe a way to test the model with a simple OLS regression if you observed K_t, I_t and market value of firms. What would happen if you didn't include taxation rules into the model, or if assumptions of part 4 did not hold?

[1] By Anton Babkin. This version: April 17, 2016.

[2] Adapted from Hayashi (1982) "Tobin's marginal q and average q: a neoclassical interpretation", Econometrica.