Econ 714: Handout 11 [1]

Credit constraints and business cycles[2]

This model presents a mechanism consistent with business cycle stylized facts without relying on a persistent exogenous TFP shock. The three business cycle characteristics of interest are:

1. Size - amplitude of fluctuations is large.
2. Persistence.
3. Asymmetry - downward movements are sharper and quicker than upward ones.

Firm's capital X fully depreciates between periods and is used in production technology $F(X)$, $F' > 0, F'' < 0$. Firm can borrow at exogenous borrowing rate r up to a limit $B_{t+1} \leq \bar{B}$. Utility is discounted at rate $\beta = 1/(1+r)$.

Firm's problem:

$$\max_{C,X,B} \sum \beta^t \ln(C_t)$$
$$Y_t = F(X_t)$$
$$\text{s.t. } C_t + X_{t+1} + B_t(1+r) = Y_t + B_{t+1}$$
$$B_{t+1} \leq \bar{B}$$
$$C_t, X_t \geq 0$$
$$X_0, B_0 \text{ given}$$

Rewrite in Bellman form with Lagrange multiplier λ_t:

$$V(X_t, B_t) = \max_{X_{t+1}, B_{t+1}} \ln(F(X_t) + B_{t+1} - X_{t+1} - B_t(1+r)) + \lambda_t(\bar{B} - B_{t+1}) + \beta V(X_{t+1}, B_{t+1})$$

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[2] These notes present a simplified version of a model from Kocherlakota (2000) "Creating business cycles through credit constraints".

Derive Euler equations:

$$\text{FOC:} \quad X_{t+1}: -\frac{1}{C_t} + \beta V_X(t+1) = 0$$

$$B_{t+1}: \frac{1}{C_t} + \beta V_B(t+1) - \lambda_t = 0$$

$$\text{ENV:} \quad X_t: V_X(t) = \frac{1}{C_t} F_X(t)$$

$$B_t: V_B(t) = -\frac{1}{C_t}(1+r)$$

$$\text{EE:} \quad \frac{1}{C_t} = \beta \frac{1}{C_{t+1}} F_X(X_{t+1}) \qquad (EE_X)$$

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}}(1+r) + \lambda_t \qquad (EE_B)$$

We are interested in seeing how GDP Y_t would respond to an unexpected shock to initial wealth $F(X_0) - B_0(1+r)$.

Let's start by characterizing the steady state equilibrium, $C_t = C, X_t = X, B_t = B$. It follows from (EE_B) and $\beta(1+r) = 1$ that in steady state $\lambda_t = 0$, i.e. borrowing constraint is never binding.

(EE_X) becomes $F_X(X_{t+1}) = 1+r$, so it can be solved for steady state X. Marginal product of capital is equal to the exogenous rate of return.

Budget constraint in steady state with $B_t = B_0 = B$ can be solved for C:

$$C + X + rB = F(X)$$

If the economy starts in steady state $X_0 = X, B_0 = B \leq \bar{B}$, it will stay there forever. Notice that it is possible to have $B_0 = \bar{B}$. If this can still be a steady state equilibrium, borrowing constraint $B_{t+1} \leq \bar{B}$ is "barely" binding.

Suppose that the economy starts at steady state and there is an additive exogenous shock Δ to the initial wealth $F(X_0) - B_0(1+r)$.

If the shock is positive, extra resources could be used to increase capital stock X_1, but that would not be optimal because capital is already at the optimal level. Instead, debt level B_1 will be reduced, so that interest payments rB_t will be smaller, and consumption C_t higher. No change in X_t implies no change in Y_t: GDP does not respond to a positive wealth shock.

If the shock is negative, but not to big, then debt can be increased without hitting the credit constraint. Lifetime consumption will be lower, but again there will be no change in X_t or Y_t.

If negative shock is sufficiently large, to accommodate it firms will have to both increase the debt level to the limit $B_1 = \bar{B}$ and to reduce capital $X_1 < X$, implying a drop in output Y_1. Subsequent dynamics is similar to a neoclassical growth model after a negative shock to capital: output Y_t will gradually converge to the steady state, accompanied with gradual increase in consumption.

So far, the model has two desirable features: *asymmetry* (only sufficiently large negative shocks have impact on GDP) and *persistence* (temporary shock to wealth has long-lasting effect on output).

Next we will consider *amplification*, i.e. the size of the output response relative to the initial disturbance. Let's define it as $A \equiv \frac{Y_1 - Y}{\Delta}$. This analysis is easier to perform on the linearized version of the model. Assume $F(X) = X^\alpha$.

Suppose that the economy starts at the steady state, and $B_0 = \bar{B}$. As the borrowing level is at the limit, a small negative shock will drive the economy into a downturn. Borrowing will always be at the limit, and capital, output and consumption will gradually converge back to the steady state. The dynamic behavior of the system will be described by the Euler equation (EE_X), budget constraint with constant $B_t = B = \bar{B}$ and initial condition X_0:

$$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} \alpha X_{t+1}^{\alpha-1}$$
$$C_t + X_{t+1} + rB = X_t^\alpha + \epsilon_t$$
$$X_0 = X$$

where $\epsilon_0 = \Delta$ and $\epsilon_t = 0, t > 0$.

Log-linearize around the steady state using lower-case variable names for percentage deviations, $x_t \approx \frac{X_t - X}{X}$ etc.

$$c_t = c_{t+1} + (1-\alpha)x_{t+1}, \quad \forall t$$
$$\frac{C}{X}c_t + x_{t+1} = \frac{1}{\beta}x_t, \quad t > 0$$
$$\frac{C}{X}c_0 + x_1 = \frac{1}{\beta}x_0 + \frac{\Delta}{X}$$
$$x_0 = 0$$

This is a system of second order linear difference equations with initial conditions. We can turn it into a single second-order difference equation by substituting out c_t.

$$x_{t+2} + (-1 - \frac{1}{\beta} - \frac{C}{X}(1-\alpha))x_{t+1} + \frac{1}{\beta}x_t = 0, \quad t > 0 \tag{1}$$
$$x_2 + (-1 - \frac{1}{\beta} - \frac{C}{X}(1-\alpha))x_1 + \frac{\Delta}{X} = 0 \tag{2}$$
$$x_0 = 0$$

(1) is a homogenous second order difference equation. It's charactristic equation $z^2 + (-1 - \frac{1}{\beta} - \frac{C}{X}(1-\alpha))z + \frac{1}{\beta} = 0$ has roots

$$z = \frac{m \pm \sqrt{m^2 - 4/\beta}}{2}$$

where $m = 1 + \frac{1}{\beta} + \frac{C}{X}(1-\alpha)$.

Roots are real and distinct, in which case the general solution of equation (1) is $x_t = \kappa_1 z_1^t + \kappa_2 z_2^t$, where κ_1 and κ_2 are constants pinned down by initial condition.

One can show that $0 < z_1 < 1$ and $z_2 > 1$. We are only interested in the stable solution, so we set $\kappa_2 = 0$. Let $\gamma \equiv z_1 < 1$. Then solution is characterized by $x_{t+1} = \gamma x_t, \quad t > 0$. Substitute this back to (2) to solve for $x_1 = \beta\gamma\frac{\Delta}{X}$.[3]

Log-linearizing $Y_t = X_t^\alpha$ yields $y_t = \alpha x_t$, and in steady state $1 = \beta\alpha\frac{Y}{X}$, so

$$y_1 = \alpha\beta\gamma\frac{\Delta}{X} = \gamma\frac{\Delta}{Y}$$

Going back to the definition of amplification ratio:

$$A \equiv \frac{Y_1 - Y}{\Delta} = \frac{Y_1 - Y}{Y}\frac{Y}{\Delta} = y_1\frac{Y}{\Delta} = \gamma$$

So $A = \gamma < 1$, i.e. there is a less than one unit change in GDP in response to a one unit shock to initial wealth. The paper also shows that if $\bar{B} \to 0$, then $A \to \alpha$ and if $\bar{B}$ gets big enough, then $A \to 1$. But there is no amplification in a model with exogenous borowing limit $\bar{B}$.

However in a full version of the model, presented in Kocherlakota (2000), $A > 1$, i.e. there is an amplification of the initial shock. Check the paper for details, but here is the intuition.

Output is produced using capital and land, $Y = F(X, L)$. X and L are complementary. Land L is in limited supply and can be traded at price Q. Suppose that the borrowing limit is $B_{t+1} \leq Q_t L_t$. This can be interpreted as collateral constraint: firms cannot borrow more than the market value of their non-depreciating assets.

If there is a negative shock to initial wealth, demand for capital X^d falls. As land is a complementary input, demand for land L^d also falls. Supply of land is fixed, so equilibrium land price Q falls. If the firm was at the limit of it's collateral constraint, tightening of the constraint means that it has to borrow less. But this is equivalent to a negative wealth shock, so loop starts over: $B \Downarrow \to X \Downarrow \to Q \Downarrow \to B \Downarrow \to$ ....

[3] This is shown in equation (43) in Kocherlakota (2000), but I could not derive it myself.