Econ 714: Final exam - Solution[1]

1

The correct budget constraint for this problem must be, in nominal terms:

$$P_t c_t + M_t + \frac{\alpha_{Bt}}{I_t} + P_t \frac{\alpha_{bt}}{R_t} + P_t S_t \alpha_{St} = \tau_{t-1} + M_{t-1} + \alpha_{Bt-1} + P_t \alpha_{bt-1} + P_t (Y_t + S_t) \alpha_{St-1}$$

where $\tau_t = M_t^s - M_t$ is nominal transfer/tax from changing money supply.

At time t decisions are made over t-indexed variables, and $t-1$ variables are states resulting from the previous period.

In real terms:

$$c_t + m_t + \frac{\alpha_{Bt}}{P_t I_t} + \frac{\alpha_{bt}}{R_t} + S_t \alpha_{St} = \frac{\tau_{t-1}}{P_t} + m_{t-1} \frac{P_{t-1}}{P_t} + \frac{\alpha_{Bt-1}}{P_t} + \alpha_{bt-1} + (Y_t + S_t) \alpha_{St-1}$$

(a) With Lagrange multiplier on budget constraint λ_t, first order conditions are:

$$\begin{aligned}
[c_t] &: \beta^t u'(c_t) = \lambda_t \\
[\alpha_{bt}] &: \frac{\lambda_t}{R_t} = \mathbb{E}_t \lambda_{t+1} \\
[\alpha_{Bt}] &: \frac{\lambda_t}{P_t I_t} = \mathbb{E}_t \lambda_{t+1} \frac{1}{P_{t+1}} \\
[\alpha_{St}] &: \lambda_t S_t = \mathbb{E}_t \lambda_{t+1} (Y_{t+1} + S_{t+1}) \\
[m_t] &: \beta^t v'(m_t) - \lambda_t + \mathbb{E}_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0
\end{aligned}$$

Substitute out λ_t and plug in goods market clearing $c_t = Y_t$ to obtain equilibrium pricing conditions:

$$\begin{aligned}
\frac{1}{R_t} &= \beta \mathbb{E}_t \frac{u'(Y_{t+1})}{u'(Y_t)} \\
\frac{1}{I_t} &= \beta \mathbb{E}_t \frac{u'(Y_{t+1})}{u'(Y_t)} \frac{P_t}{P_{t+1}} \\
S_t &= \beta \mathbb{E}_t \frac{u'(Y_{t+1})}{u'(Y_t)} (Y_{t+1} + S_{t+1}) \\
1 - \frac{v'(m_t)}{u'(Y_t)} &= \beta \mathbb{E}_t \frac{u'(Y_{t+1})}{u'(Y_t)} \frac{P_t}{P_{t+1}} = \frac{1}{I_t}
\end{aligned}$$

[1]By Anton Babkin. March 14, 2016.

(b) Endowment process is $\frac{Y_{t+1}}{Y_t} = \exp(\mu + \sigma W_{t+1})$. Using real bond pricing equation:

$$\begin{aligned}
\frac{1}{R_t} &= \beta \mathbb{E}_t \left(\frac{Y_{t+1}}{Y_t} \right)^{-\gamma} \\
&= \beta \mathbb{E}_t \exp(-\gamma\mu - \gamma\sigma W_{t+1}) \\
&= \beta \exp(-\gamma\mu + \gamma^2\sigma^2/2) \\
-\log R_t &= \log\beta - \gamma\mu + \gamma^2\sigma^2/2 \\
r_t &= \gamma\mu - \gamma^2\sigma^2/2 - \log\beta
\end{aligned}$$

Real bonds return positively depends on growth rate μ and negatively on volatility σ and patience β. Effect of γ is ambiguous.

(c)

$$\begin{aligned}
\pi_t &\equiv \log \mathbb{E}_t \frac{P_{t+1}}{P_t} \\
&= \log \mathbb{E}_t \left(\frac{Y_{t+1}}{Y_t} \right)^{a} \\
&= \log \mathbb{E}_t \exp(a\mu + a\sigma W_{t+1}) \\
&= a\mu + a^2\sigma^2/2
\end{aligned}$$

If $\sigma = 0$, then simply $\pi_t = a\mu$. If $\sigma > 0$, this is a quadratic equation in a which generally has two roots:

$$a = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2\pi_t}}{\sigma^2} \tag{1}$$

(d) Using nominal bond pricing equation and solution of class $P_t = Y_t^a$:

$$\begin{aligned}
\frac{1}{I_t} &= \beta \mathbb{E}_t \left(\frac{Y_{t+1}}{Y_t} \right)^{-\gamma-a} \\
&= \beta \exp(-(\gamma+a)\mu + (\gamma+a)^2\sigma^2/2) \\
-\log I_t &= \log\beta - (\gamma+a)\mu + (\gamma+a)^2\sigma^2/2 \\
i_t &= (\gamma+a)\mu - (\gamma+a)^2\sigma^2/2 - \log\beta \\
&= \gamma\mu - \gamma^2\sigma^2/2 - \log\beta + a\mu + a^2\sigma^2/2 - \gamma a\sigma^2 - a^2\sigma^2 \\
&= r_t + \pi_t - \gamma a\sigma^2 - a^2\sigma^2
\end{aligned}$$

where a is given by the equation (1).

(e) Without risk, $\sigma = 0$, Fisher equation holds exactly and in unique equilibrium $\pi_t = \bar{i}_t - r_t$, $a = \pi_t/\mu$.

If $\sigma > 0$, there might be two possible inflation levels in equilibrium that correspond to the two roots for a.

In these equilibria inflation is a function of the endowment growth, so return on the real bond is correlated with inflation. Decomposition of nominal interest rate into real interest rate and inflation (Fisher equation) now includes an additional covariance term - inflation risk - that can take two values for different a:

$$\begin{aligned}
\frac{1}{I_t} &= \beta \mathbb{E}_t \frac{u'(Y_{t+1})}{u'(Y_t)} \frac{P_t}{P_{t+1}} \\
&= \mathbb{E}_t \beta \frac{u'(Y_{t+1})}{u'(Y_t)} \mathbb{E}_t \frac{P_t}{P_{t+1}} + Cov_t \left(\beta \frac{u'(Y_{t+1})}{u'(Y_t)}, \mathbb{E}_t \frac{P_t}{P_{t+1}} \right) \\
&= \frac{1}{R_t} \mathbb{E}_t \frac{P_t}{P_{t+1}} + Cov_t \left(\beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-\gamma}, \left(\frac{Y_{t+1}}{Y_t} \right)^{-a} \right)
\end{aligned}$$

2

(a)

$$\begin{aligned}
rW &= w + \lambda(U(s) - W) \\
rU(s) &= z - c(s) + q(s)(W - U(s))
\end{aligned}$$

Solve for $W - U = \frac{w-z+c(s)}{r+\lambda+q(s)}$. Substitute back to get steady state values of $U(s)$ and W:

$$\begin{aligned}
rW &= w - \lambda \frac{w - z + c(s)}{r + \lambda + q(s)} \\
rU(s) &= z - c(s) + q(s) \frac{w - z + c(s)}{r + \lambda + q(s)}
\end{aligned}$$

(b) First order condition of the unemployed with respect to s:

$$c'(s) = q'(s)(W - U) = q'(s) \frac{w - z + c(s)}{r + \lambda + q(s)} \tag{2}$$

(c) Rewrite (2) as

$$\frac{c'(s)(r + \lambda)}{q'(s)} + c'(s)s - c(s) = w - z$$

Derivative of the LHS with respect to s is

$$\begin{aligned}
(r + \lambda) & \frac{c''(s)q'(s) - c'(s)q''(s)}{(q'(s))^2} + c''(s)s + c'(s) - c'(s) \\
&= (r + \lambda) \frac{c''(s)q'(s) - c'(s)q''(s)}{(q'(s))^2} + c''(s)s
\end{aligned}$$

and is positive since $c''(s) > 0$ and $q''(s) < 0$.

So the LHS is increasing in s and the RHS is constant. So s increases as w increases.

3

(a) When price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, two things happen:

- The relative price of firms who have not set their price for a while falls, they experience an increase in demand and raise output. Firms who have just reset their prices reduce output. This production dispersion is inefficient.
- Consumers increase consumption of the goods whose relative price has fallend a reduce cunsumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.

(b) Risk premium of a risky return over a risk-free one can be expressed as

$$\frac{E(r_t) - r^f}{\sigma(r)} = \gamma\sigma(\Delta c_t)corr(\Delta c_t, r_t)$$

A puzzle is that empirical estimates of this equation imply that γ needs to be about 27. This is a puzzle, because such high levels of risk-aversion imply implausibly high premiums invididviduals will be willing to pay to avoid taking lotteries with zero expected payoff. Existing micro-level studies suggest that γ should be in the order of 2 to 3.

But even if we allow risk-aversion to be very high, it won't resolve the puzzle. With CRRA utility γ is not only a coefficient of relative risk-aversion, but also an inverse of the elastisity of intertemporal substitution. For an observed levels of aggregate consumption growth, this implies that risk-free rate must be much higher than it historically was.

(c) Time consistency problem arises when future plans that are optimal at a particular point in time become not optimal when that future actually comes.

In Ramsey problem, it is socially optimal to set zero tax on returns to capital for all future periods except the initial one, because then capital is already in place, and proportional tax effectively becomes a lump sum tax and does not distort households' incentives. However if the planner is allowed to reoptimize at some time in the future, he would choose to deviate from the zero-tax plan and tax capital in that period.