Noah Williams
Department of Economics
University of Wisconsin

Economics 714
Macroeconomic Theory
Spring 2016

Problem Set 1

Due in Class on 2/10

1. In the lecture notes we gave (without proof) the following characterization of the reservation wage in an environment with perfect job finding ($p = 1$) and no separations ($s = 0$).
$$w_R - z = \beta(E[w] - z) + \beta \int_0^{w_R} F(w)dw$$
   (a) Derive the analogue of this condition in an environment with imperfect job finding ($p < 1$) and separations ($s > 0$).

   (b) Prove that if an offer distribution G is a mean preserving spread of F then the reservation wage is greater for G than F. Note that we can suppose that both F and G have finite support $[0, \bar{w}]$, and they share the same mean so $\int_0^{\bar{w}} wdF(w) = \int_0^{\bar{w}} wdG(w)$. But G is a mean-preserving spread of F, which we can characterize as $\int_0^b [G(w) - F(w)]dw \geq 0$ for $0 \leq b \leq \bar{w}$.

   (c) Prove that if the job offer rate p falls then the steady state unemployment rate increases, even though the reservation wage falls.

2. Consider a search model with heterogeneous jobs, where employed workers have the option to search for better jobs. Unemployed workers receive income z, and find jobs with Poisson rate f. All new jobs start at the highest productivity of 1, but with Poisson rate λ a productivity shock arrives, resulting in a new productivity x drawn from a distribution G with support on $[0, 1]$. If the productivity is below a threshold R, the job is destroyed and employed workers become unemployed. In addition, employed workers have the option to search for new jobs. If they pay a cost σ they can search, which yields a new job (again at $x = 1$) at the same rate f as for unemployed workers. Wages of employed workers $w(x)$ differ according to their job, but suppose that they do not depend on whether the worker searches or not. Write down the Hamilton-Jacobi-Bellman equations determining the following values: U of an unemployed worker, $W^n(x)$ of a worker employed at a job with productivity x who chooses not to search, and $W^s(x)$ of a worker employed at a job with productivity x who chooses to search for a new job. Note that when a productivity shock arrives, a worker may wish to change his decision of whether to search or not.

3. An economy consists of two types of consumers indexed by $i = 1, 2$. There is one nonstorable consumption good. Let (e_t^i, c_t^i) be the endowment, consumption pair for consumer i in period t. Consumer 1 has endowment stream $e_t^1 = 1$ for $t = 0, 1, ..., 20$, and $e_t^1 = 0$ for $t \geq 21$. Consumer 2 has endowment stream $e_t^2 = 0$ for $0 \leq t \leq 20$ and

$y_t^2 = 1$ for $t \geq 21$. Both consumers have preferences ordered by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i)$$

where u is increasing, twice differentiable, and strictly concave.

(a) Let the Pareto weight on consumer 1 be $\lambda \in (0,1)$ and the weight on consumer 2 be $1-\lambda$. Compute the Pareto optimal allocation.

(b) Define a competitive equilibrium for this economy (with trading at date 0).

(c) Compute a competitive equilibrium. Interpret your results, relating them to the Pareto optima in part (a).

(d) What are the equilibrium prices of the following assets:

  i. A claim to consumer 1's endowment process.

  ii. A claim to consumer 2's endowment process.

  iii. A claim to the aggregate endowment process.

4. An economy consists of two types of consumers indexed by $i = 1, 2$. There is one nonstorable consumption good. Let (e_t^i, c_t^i) be the endowment, consumption pair for consumer i in period t. Both consumers have preferences ordered by

$$U^i = E \sum_{t=0}^{\infty} \beta^t \log(c_t^i).$$

The endowment streams of the two consumers are governed by two independent 2-state Markov chains $s_t \in \{0,1\}$ with transition matrix P_s and $a_t \in \{0,1\}$ with transition matrix P_a, where $P_{s,12} = \Pr\{s_{t+1} = 1 | s_t = 0\}$ and so on. Suppose that each chain is stationary and that the initial states (s_0, a_0) are drawn from the stationary distribution, so that $\Pr\{s_0 = 0\} = \bar{\pi}_s$ and $\Pr\{a_0 = 0\} = \bar{\pi}_a$ Consumer 1 has endowment $e_t^1 = a_t + s_t$, while consumer 2 has endowment $e_t^2 = a_t + 1 - s_t$.

(a) Write out explicitly the preferences U^i of a consumer in terms of the history of the states and their associated probabilities.

(b) Define a competitive equilibrium for this economy.

(c) Characterize the competitive equilibrium for this economy, calculating the prices of all Arrow-Debreu securities. How does the allocation vary across the (s_t, a_t) states? Across time? Across consumers?

(d) Now consider an economy with sequential trading in Arrow securities, one-period ahead claims to contingent consumption. How many Arrow securities are there? Compute their prices in the special case $\beta = 0.95, P_{s,11} = 0.9, P_{s,22} = 0.8, P_{a,11} = 0.8, P_{a,22} = 0.7$.

(e) Using these same parameters, in each state what is the price of a one-period ahead riskless claim to one unit of consumption?