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Economics 714
Macroeconomic Theory
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Problem Set 3

Due in Class on 3/7

1. Consider the following two period model with a unit measure of agents. Agents are endowed in period 1 with y_1 units of the consumption good, which can be consumed as c_1 or stored (at a gross interest rate of unity). In the second period half of the agents are highly skilled and one unit of their labor generates θ_H units of consumption, and the other half are low skilled and one unit of labor gives $\theta_L < \theta_H$ units of consumption. Agents have identical utility functions of the form

$$E_1[u(c_1) + \beta[u(c_2) - v(l)]].$$

where the expectation is over the skill realizations $\theta_i \in \{\theta_H, \theta_L\}$. Consider the problem of a social planner that observes each agent's production $x = \theta_i l$ but does not separately observe productivity and hours. The planner chooses an allocation $\{c_1(\theta), c_2(\theta), x(\theta)\}$ that maximizes utility subject to the feasibility condition that total consumption equal total output, and the incentive constraints that each agent truthfully report his skill realization.

   (a) Write out explicitly the problem of the social planner and the relevant constraints.

   (b) Suppose that at the beginning of the first period agents learn their second-period skills, so that there is no uncertainty. Show that an optimal allocation leads to no distortion of each agent's consumption, i.e. that a standard Euler equation holds.

   (c) Show that in an optimal allocation the labor supply decision of a high skill agent is undistorted, i.e. that his marginal rate of substitution between consumption and leisure is equal to his marginal product.

   (d) Now suppose that at the beginning of the first period agents receive a signal $\alpha \in \{\alpha_H, \alpha_L\}$ of their future productivity. The signal is informative but not completely so: $1 > P(\theta = \theta_H | \alpha = \alpha_H) > 0.5$. In the allocation, consumption is the first period is thus conditional on the report of α, but consumption in the second period and labor are conditional on α and θ. Now show that it is optimal to distort each agent's intertemporal consumption allocation.

2. Consider a cash in advance version of a Lucas model, where the household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t is a good that must be purchased with cash denominated in dollars. The endowment d_t follows a Markov process with transition function $Q(d, d')$. In addition the government prints money M_t in a stochastic manner, with the growth rate of money $g_t = M_{t+1}/M_t$ following a Markov process with transition function $F(g, g')$. Define $x_t = (d_t, g_t)$. Each period t, after shocks are realized the asset market opens first allowing the household to trade in one-period risk-free nominal bonds which cost \$1 today and pay off $\$R(x_t)$ in the next period, claims to the dividend process (trees) with (real) price $p(x_t)$, and currency M_t^d. The household enters this market with an amount s_{t-1} of trees, M_{t-1}^d in currency, and B_{t-1} in nominal bonds carried over from the previous period, and receives a real lump sum transfer τ_t, which may be positive or negative, from the government. At the same time, the government prints (or retires) money, with the the proceeds providing the funds for the transfer. Next the goods market opens and the household uses its cash for purchase of consumption goods, thus the cash in advance constraint (CIA) must hold:

$$P_t c_t \leq M_t^d,$$

So the government budget constraint is:

$$\tau_t = \frac{M_{t+1} - M_t}{P_t},$$

while the (real) household budget constraint is:

$$\frac{M_t^d}{P_t} - \tau_t + p(x_t)s_t + \frac{B_t}{P_t} \leq \left(d_{t-1}\frac{P_{t-1}}{P_t} + p(x_t)\right)s_{t-1} + \frac{B_{t-1}R(x_t)}{P_t} + \frac{M_{t-1}^d - P_{t-1}c_{t-1}}{P_t}$$

where the final term is the cash carried over from the previous period. Nominal bonds are in zero net supply, there is a single tree in the economy, and in equilibrium the household must hold all the cash printed by the government.

(a) Assume that in equilibrium $R(x_t) > 1$. (What does this imply about the CIA?) Defining w_t as household wealth at date t (the right side of the budget constraint), write down the household's Bellman equation, paying special attention to the choice of state variables and laws of motion. What conditions ensure that this equation is solvable? What conditions ensure that the value function is increasing and differentiable in wealth?

(b) Define a competitive equilibrium in this environment.

(c) Derive the household optimality conditions and find the Euler equations determining the pricing function $p(x_t)$ and interest rate $R(x_t)$.

For the next parts, suppose that $u(c) = \log c$ (don't worry if this does not satisfy your conditions above) and that money growth follows:

$$\frac{1}{g_{t+1}} = \rho\frac{1}{g_t} + \epsilon_{t+1}$$

where $0 < \rho < 1$ and ϵ_{t+1} is i.i.d. with bounded support $[\underline{\epsilon}, \overline{\epsilon}]$ with $\overline{\epsilon} < (1 - \rho)/(\beta\rho)$. Assume as well that $g_0 \in [\frac{\underline{\epsilon}}{1-\rho}, \frac{\overline{\epsilon}}{1-\rho}]$, so g_t remains in that same interval for all t.

(d) Find the equilibrium return $R(x_t)$ and verify $R(x_t) > 1$.

(e) Find the equilibrium pricing function $p(x_t)$.

(f) Describe how an increase in the growth rate of money affects inflation and stock (tree) prices.

3. Consider a simple search model of the housing market. Time is continuous, and there is a population of agents of mass 1 who are risk neutral and discount at rate r. A fraction H of the population owns (indivisible) houses, which are identical and yield flow utility u to their owners each instant. The fraction $(1 - H)$ of people without houses get no flow utility, but search for a house. Agents in the economy meet each other with Poisson arrival rate α. When a potential buyer meets a potential seller, they trade the house at price p (which they take as given) with probability $\pi = \pi_0\pi_1$, where π_0 is the buying probability and π_1 is the selling probability.

   (a) Write down the (Hamilton-Jacobi) Bellman equations determining V_1, the value of the owner of a house, and V_0, the value of an agent searching to buy a house. What is the welfare gain of owning a house $V_1 - V_0$?

   (b) Characterize the optimal buying and selling strategies which determine π_0 and π_1. Assume that agents trade when they are indifferent.

   (c) Find the equilibrium price p. How does it depend on the housing supply H?