

**Macro Example Comp Question from Quarter 1, 2015:**

Analyze the dynamic equilibrium of a two period lived overlapping generations production economy with a constant unit measure of identical agents in each generation. Each agent's preferences are given by the utility function

$$u(c_{t+1}^t, n_t^t) = \frac{1}{1-\gamma} (c_{t+1}^t)^{1-\gamma} - n_t^t$$

where  $c_{t+1}^t$  denotes period  $t+1$  consumption of an agent born in period  $t$  (i.e. an old person),  $n_t^t \in [0, 1]$  denotes labor supplied in period  $t$  by an agent born in period  $t$  (i.e. a young person). and  $\gamma \in (0, 1)$ . Technology is given by the production function  $f(k_t, n_t) = k_t^{1/2} n_t^{1/2}$  where  $k_t$  is total capital available to each producer in period  $t$ . Capital fully depreciates after production, so  $k_{t+1} = i_t$  where  $i_t$  is investment chosen by the young in period  $t$  from their labor earnings. Young agents born in period  $t$  supply labor  $n_t^t$  at real wage  $w_t$  in order to buy capital  $k_{t+1}$  which they rent to firms in their second period of life at real gross return  $R_{t+1}$  to obtain funds  $R_{t+1}k_{t+1}$  for purchasing consumption goods  $c_{t+1}^t$ .

1. Write down the optimization problem faced by a generation  $t$  agent. Solve for labor supply and investment decision rules. (5 points)
2. Write down the optimization problem faced by a representative firm which rents labor at price  $w_t$  and capital at gross rate  $R_t$  to maximize real profits. (2.5 points)
3. Define a competitive equilibrium. (2.5 points)
4. Show that a competitive equilibrium satisfies the following pair of first order difference equations

$$\begin{aligned} k_{t+1} &= R_t k_t, \\ R_{t+1}^{1-\gamma} &= 4k_t^\gamma R_t^{1+\gamma}. \end{aligned}$$

(10 points)

5. Describe stationary competitive equilibria in  $(k_t, R_t)$  space and investigate their stability. (12.5 points)
6. Suppose that the economy starts in a non-trivial steady state. Using a phase diagram, describe the dynamics of  $(k_t, R_t)$  after an unexpected negative shock to the capital stock (i.e. part of the capital accumulated by the young cohort is destroyed when they become old). Further, what happens to wages, labor supply, and consumption over time in response to the shock? (17.5 points)